

M1 MAY 2012

1. Two particles A and B , of mass $5m$ kg and $2m$ kg respectively, are moving in opposite directions along the same straight horizontal line. The particles collide directly. Immediately before the collision, the speeds of A and B are 3 m s^{-1} and 4 m s^{-1} respectively. Immediately after the collision, the direction of motion of A is unchanged by the collision. Immediately after the collision, the speed of A is 0.8 m s^{-1} .

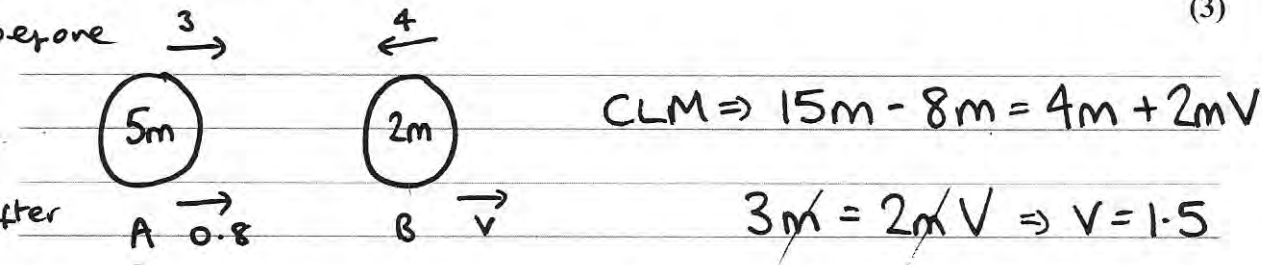
- (a) Find the speed of B immediately after the collision.

(3)

In the collision, the magnitude of the impulse exerted on A by B is 3.3 N s .

- (b) Find the value of m .

(3)



b) Mom A before = $15m$ \therefore Impulse = $11m$
Mom A after = $4m$

$11m = 3.3 \Rightarrow m = 0.3$

2.

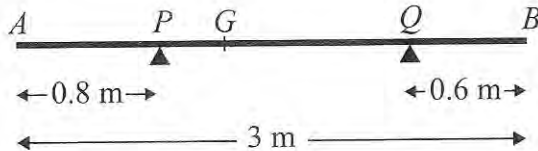
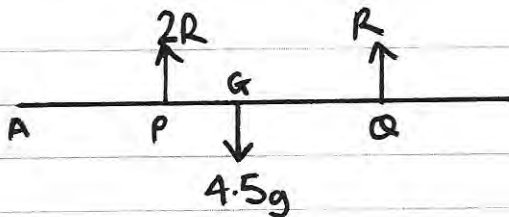


Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q , where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G . Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod, (3)

(b) the distance AG . (4)



$$R \uparrow = \downarrow \quad 3R = 4.5g$$

$$\therefore R = \underline{1.5g \text{ N}}$$

$$\text{b) } \overset{A}{\curvearrowright} \quad 4.5g \times AG = 3g \times 0.8 + 1.5g \times 2.4$$

$$4.5g \times AG = 6g \quad \Rightarrow AG = \frac{4}{3}$$

3.

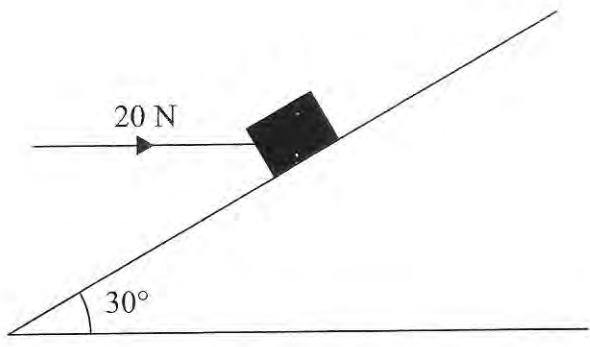
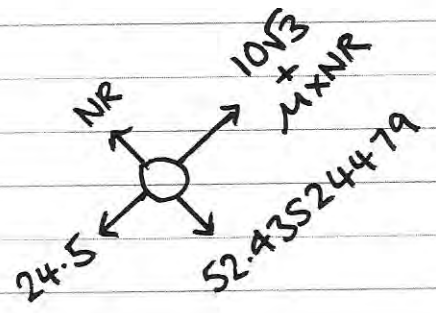
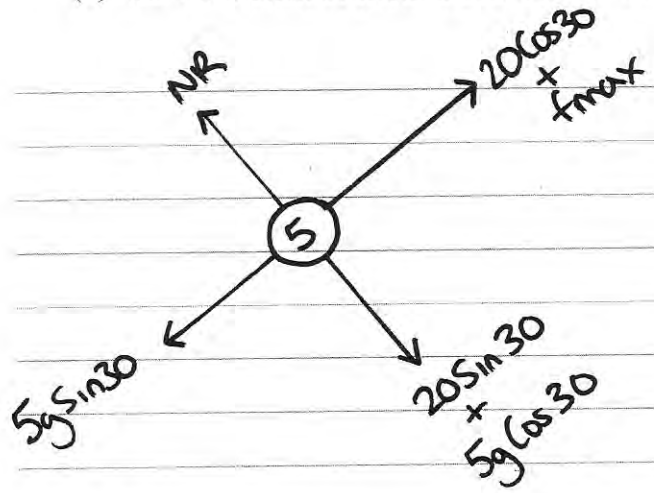


Figure 2

A box of mass 5 kg lies on a rough plane inclined at 30° to the horizontal. The box is held in equilibrium by a horizontal force of magnitude 20 N, as shown in Figure 2. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The box is in equilibrium and on the point of moving down the plane. The box is modelled as a particle.

Find

- (a) the magnitude of the normal reaction of the plane on the box, (4)
- (b) the coefficient of friction between the box and the plane. (5)



$$R_{f\uparrow} = 0 \Rightarrow NR = 52.4 \text{ (3sf)}$$

$$R_{f\uparrow} = 0 \quad 10\sqrt{3} + \mu(52.43\dots) = 24.5$$

$$\mu = 0.137 \text{ (3sf)}$$

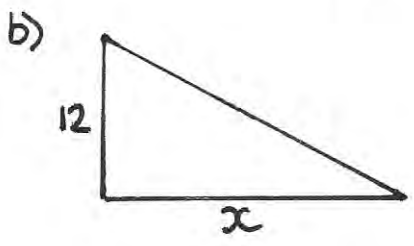
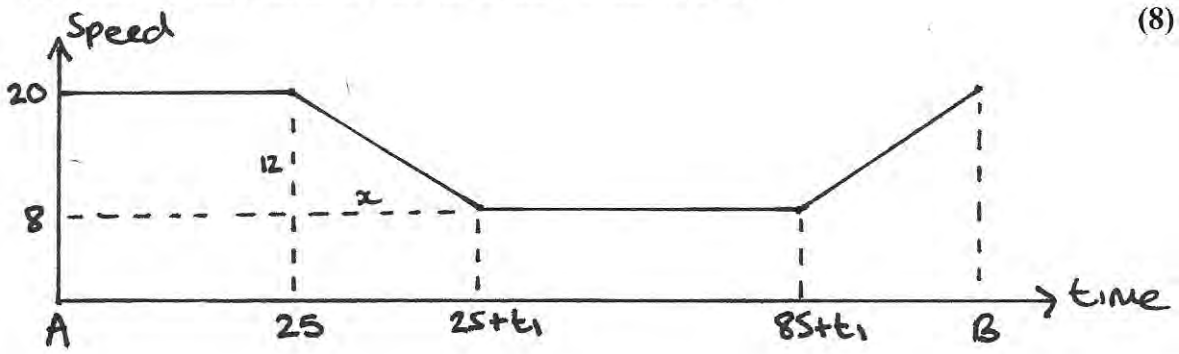
4. A car is moving on a straight horizontal road. At time $t = 0$, the car is moving with a speed of 20 m s^{-1} and is at the point A . The car maintains the speed of 20 m s^{-1} for 25 s, then moves with constant deceleration 0.4 m s^{-2} , reducing its speed from 20 m s^{-1} to 8 m s^{-1} . The car then moves with constant speed 8 m s^{-1} for 60 s. The car then moves with constant acceleration until it is moving with speed 20 m s^{-1} at the point B .

(a) Sketch a speed-time graph to represent the motion of the car from A to B . (3)

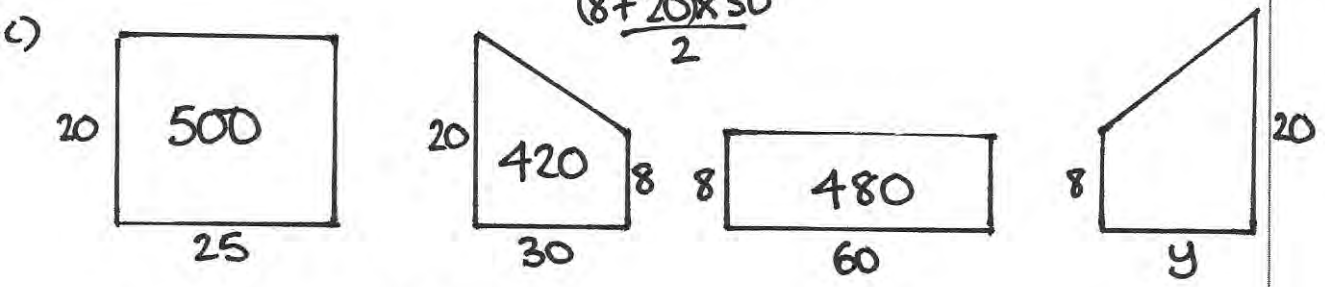
(b) Find the time for which the car is decelerating. (2)

Given that the distance from A to B is 1960 m,

(c) find the time taken for the car to move from A to B . (8)



gradient = $-0.4 = -\frac{12}{x} \Rightarrow x = \frac{12}{0.4}$
 $x = 30 \text{ sec}$



$500 + 420 + 480 = 1400$
 $1960 - 1400 = 560$

$\therefore \frac{(8+20) \times y}{2} = 560$

$28y = 1120$
 $y = 40$

$\therefore \text{total time} = 25 + 30 + 60 + 40$
 $= \underline{155 \text{ sec}}$

5. A particle P is projected vertically upwards from a point A with speed u m s⁻¹. Point A is 17.5 m above horizontal ground. The particle P moves freely under gravity and reaches the ground with speed 28 m s⁻¹.

(a) Show that $u = 21$

(3)

At time t seconds after projection, P is 19 m above A .

(b) Find the possible values of t .

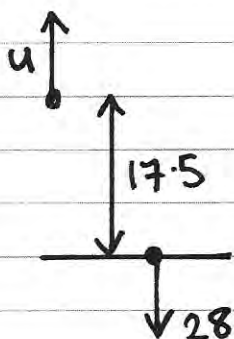
(5)

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P .

(c) Find the vertical distance that P sinks into the ground before coming to rest.

(4)

a)



$$\begin{aligned} s &= -17.5 \\ u & \\ v &= -28 \\ a &= -9.8 \\ t & \end{aligned}$$

$$v^2 = u^2 + 2as$$

$$784 = u^2 + 343$$

$$u^2 = 441 \Rightarrow u = 21 \quad \#$$

b)

$$s = 19$$

$$u = 21$$

$$v$$

$$a = -9.8$$

$$t$$

$$s = ut + \frac{1}{2}at^2$$

$$19 = 21t - 4.9t^2$$

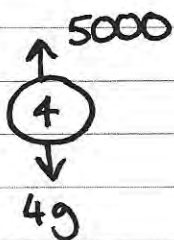
$$4.9t^2 - 21t + 19 = 0$$

$$t = \frac{21 \pm \sqrt{21^2 - 4(4.9)(19)}}{9.8}$$

$$t_1 = 2.99 \text{ (3sf)}$$

$$t_2 = 1.30 \text{ (3sf)}$$

c)



$$Rf \downarrow = ma \Rightarrow 39.2 - 5000 = 4a$$

$$\Rightarrow a = -1240.2$$

$$u = 28, v = 0$$

$$v^2 = u^2 + 2as \Rightarrow 0 = 784 - 2480.4s$$

$$\Rightarrow s = 0.316 \text{ m (3sf)}$$

6. [In this question \mathbf{i} and \mathbf{j} are horizontal unit vectors due east and due north. Position vectors are given with respect to a fixed origin.]

A ship S is moving with constant velocity $(-12\mathbf{i} + 7.5\mathbf{j}) \text{ km h}^{-1}$.

- (a) Find the direction in which S is moving, giving your answer as a bearing.

At time t hours after noon, the position vector of S is \mathbf{s} km. When $t = 0$, $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$.

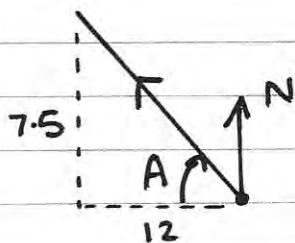
- (b) Write down \mathbf{s} in terms of t .

A fixed beacon B is at the point with position vector $(7\mathbf{i} + 12.5\mathbf{j}) \text{ km}$.

- (c) Find the distance of S from B when $t = 3$

- (d) Find the distance of S from B when S is due north of B .

a)



$$\begin{aligned} \text{bearing} &= 270 + A \\ &= 270 + \tan^{-1}\left(\frac{7.5}{12}\right) \\ &= \underline{\underline{302^\circ}} \end{aligned}$$

- b) pos after $t = \text{orig pos} + t(\text{vel})$

$$\mathbf{s} = \begin{pmatrix} 40 \\ -6 \end{pmatrix} + t \begin{pmatrix} -12 \\ 7.5 \end{pmatrix}$$

c) $t = 3 \quad \mathbf{s} = \begin{pmatrix} 40 - 36 \\ -6 + 22.5 \end{pmatrix} \quad \mathbf{s} = \begin{pmatrix} 4 \\ 16.5 \end{pmatrix}$

$$\vec{BS} = \underline{\underline{\mathbf{s}}} - \underline{\underline{\mathbf{B}}} = \begin{pmatrix} 4 \\ 16.5 \end{pmatrix} - \begin{pmatrix} 7 \\ 12.5 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$$\text{dist} = |\vec{BS}| = \sqrt{3^2 + 4^2} = 5 \text{ km.}$$

- d) due North \Rightarrow i component of $\vec{BS} = 0$ (i part of $\mathbf{s} = -7!$)
 $(40 - 12t) = 7 \quad 12t = 33 \quad t = \frac{11}{4}$

$$t = \frac{11}{4} \quad S = \left(40 - 12 \left(\frac{11}{4} \right) \right) =$$
$$-6 + 7.5 \left(\frac{11}{4} \right) =$$

$$\vec{SB} = \frac{117}{8} - 12 \cdot S = \frac{17}{8} \text{ km}$$

7.



Figure 3

Two particles P and Q , of mass 0.3 kg and 0.5 kg respectively, are joined by a light horizontal rod. The system of the particles and the rod is at rest on a horizontal plane. At time $t = 0$, a constant force F of magnitude 4 N is applied to Q in the direction PQ , as shown in Figure 3. The system moves under the action of this force until $t = 6\text{ s}$. During the motion, the resistance to the motion of P has constant magnitude 1 N and the resistance to the motion of Q has constant magnitude 2 N .

Find

(a) the acceleration of the particles as the system moves under the action of F , (3)

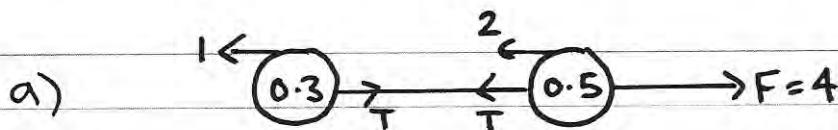
(b) the speed of the particles at $t = 6\text{ s}$, (2)

(c) the tension in the rod as the system moves under the action of F . (3)

At $t = 6\text{ s}$, F is removed and the system decelerates to rest. The resistances to motion are unchanged. Find

(d) the distance moved by P as the system decelerates, (4)

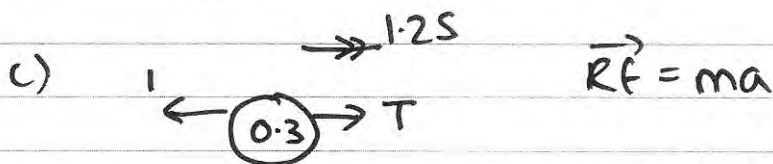
(e) the thrust in the rod as the system decelerates. (3)



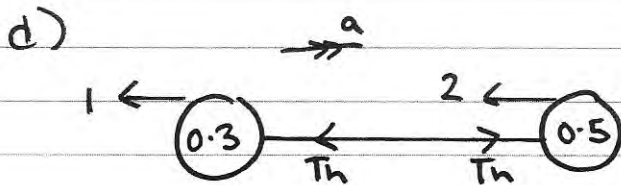
whole system $Rf = ma$ $4 - 2 - 1 = 0.8a$
 $\therefore a = \underline{1.25\text{ ms}^{-2}}$

b) $u = 0$ $t = 6$ $a = 1.25$ $v = u + at$

$v = 0 + 1.25 \times 6$ $\Rightarrow v = \underline{7.5\text{ ms}^{-1}}$



$$T - 1 = 0.3 \times 1.25 \Rightarrow T =$$



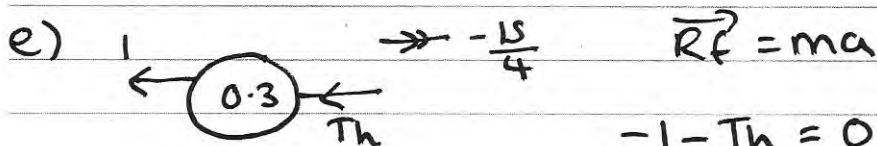
whole system $\vec{Rf} = ma$ $-1 - 2 = 0.8a$

$$\Rightarrow a = -\frac{15}{4}$$

$$u = 7.5 \quad v = 0 \quad a = -\frac{15}{4}$$

$$v^2 = u^2 + 2as \Rightarrow 0 = 86.25 - \frac{15}{2}s$$

$$\therefore s = 7.5 \text{ m}$$



$$-1 - T_h = 0.3 \left(-\frac{15}{4} \right)$$

$$-1 - T_h = -\frac{9}{8}$$

$$-T_h = -\frac{1}{8}$$

$$\therefore \text{Thrust} = \underline{\underline{\frac{1}{8} \text{ N}}}$$